Multimedia Systems

Exercise No. 2

Forward Error Correction: Hamming code

The Hamming code is a simple block code named after its inventor, Richard Hemming. It follows the scheme $(2^r-1, 2^r-1-r)$ and allows the correction of one bit error. For $r=3$ results a (7,4) Hamming code, which adds 3 controll bits to 4 coded bits. The Hamming code is a linear block code and can therefore be described in matrix notation.

Given is the following (7,4) Hamming code:

The generator matrx is given with

$$G = (I_4 \mid P) = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

The associated parity-check matrix is

$$H = (-P^T \mid I_3) = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

A **coding** can be calculated by a matrix multiplication of the source bit string $\vec{x}$ with the generator matrix: $\vec{y}=\vec{x}\cdot G$
Note: $\vec{x}$ and $\vec{y}$ are row vectors.

For **decoding,** the received bit sting is multiplicated with the transposed parity-check matrix:
$\vec{s}=\vec{r}\cdot H^T$
Depending on the calculated result $\vec{s}$ the error vector $\vec{e}$ can be determined with the following table:

$\vec{s}$			$\vec{e}$						
0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0
0	1	1	0	1	0	0	0	0	0
1	1	1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	1

The received bit string is corrected with $\vec{y}' = \vec{r} \oplus \vec{e}$. The first four bit represent the source bit sting.

1. Choose a arbitrary bit string (4 bit)

    a) Code the bit string with the given (7,4) Hamming code.
    b) Suppose there is no error during the transmission. Decode the coded bit string.
    c) During the transmission, one bit (of your choice) is inverted. Decode the received bit string.
    d) Decode the received bit string after inverting one more bit.

2. Discuss the realtionship between **error rate**, **overhead** and **complexity of decoding**.