

Multimedia Systems

Exercise No. 2

Forward Error Correction: Hamming code

The Hamming code is a simple block code named after its inventor, Richard Hemming. It follows the scheme $(2^r-1, 2^r-1-r)$ and allows the correction of one bit error. For $r=3$ results a (7,4) Hamming code, which adds 3 control bits to 4 coded bits. The Hamming code is a linear block code and can therefore be described in matrix notation.

Given is the following (7,4) Hamming code:

The generator matrix is given with

$$G = (I_4 | P) = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

The associated parity-check matrix is

$$H = (-P^T | I_3) = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

A **coding** can be calculated by a matrix multiplication of the source bit string \vec{x} with the generator matrix: $\vec{y} = \vec{x} \cdot G$

Note: \vec{x} and \vec{y} are row vectors.

For **decoding**, the received bit string is multiplied with the transposed parity-check matrix:

$$\vec{s} = \vec{r} \cdot H^T$$

Depending on the calculated result \vec{s} the error vector \vec{e} can be determined with the following table:

\vec{s}			\vec{e}						
0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0
0	1	1	0	1	0	0	0	0	0
1	1	1	0	0	1	0	0	0	0
1	0	1	0	0	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	1

The received bit string is corrected with $\vec{y}' = \vec{r} \oplus \vec{e}$. The first four bit represent the source bit string.

1. Choose a arbitrary bit string (4 bit)

- a) Code the bit string with the given (7,4) Hamming code.
- b) Suppose there is no error during the transmission. Decode the coded bit string.
- c) During the transmission, one bit (of your choice) is inverted. Decode the received bit string.
- d) Decode the received bit string after inverting one more bit.

2. Discuss the relationship between **error rate**, **overhead** and **complexity of decoding**.
